# 6.2 Homogeneous Problems in Linear Elasticity

A **homogeneous** stress (strain) field is one where the stress (strain) is the same at all points in the material. Homogeneous conditions will arise when the geometry is simple and the loading is simple.

## 6.2.1 Elastic Rectangular Cuboids

Hooke's Law, Eqns. 6.1.8 or 6.1.9, can be used to solve problems involving homogeneous stress and deformation. Hoooke's law is 6 equations in 12 unknowns (6 stresses and 6 strains). If some of these unknowns are given, the rest can be found from the relations.

### Example

Consider the block of linear elastic material shown in Fig. 6.2.1. It is subjected to an equi-biaxial stress of $\sigma_{xx} = \sigma_{yy} = \bar{\sigma} > 0$.

Since this is an isotropic elastic material, the shears stresses and strains will be all zero for such a loading. One thus need only consider the three normal stresses and strains.

There are now 3 equations (the first 3 of Eqns. 6.1.8 or 6.1.9) in 6 unknowns. One thus needs to know *three* of the normal stresses and/or strains to find a solution. From the loading, one knows that $\sigma_{xx} = \bar{\sigma}$ and $\sigma_{yy} = \bar{\sigma}$. The third piece of information comes from noting that the surfaces parallel to the $x - y$ plane are free surfaces (no forces acting on them) and so $\sigma_{zz} = 0$.

From Eqn. 6.1.8 then, the strains are

$$\varepsilon_{xx} = \varepsilon_{yy} = (1-\nu)\frac{\bar{\sigma}}{E}, \quad \varepsilon_{zz} = -2\nu\frac{\bar{\sigma}}{E}, \quad \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0$$

As expected, $\varepsilon_{xx} = \varepsilon_{yy}$ and $\varepsilon_{zz} < 0$.

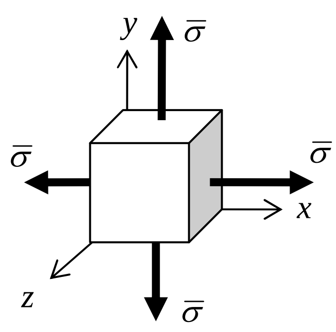


**Figure 6.2.1: A block of linear elastic material subjected to an equi-biaxial stress**

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### 6.2.2 Problems

1. A block of isotropic linear elastic material is subjected to a compressive normal stress $\sigma_o$ over two opposing faces. The material is constrained (prevented from moving) in one of the direction normal to these faces. The other faces are free.
   (a) What are the stresses and strains in the block, in terms of $\sigma_o, \nu, E$ ?
   (b) Calculate three maximum shear stresses, one for each plane (parallel to the faces of the block). Which of these is the overall maximum shear stress acting in the block?

2. Repeat problem 1a, only with the free faces now fixed also.