4.3 Volumetric Strain

The volumetric strain is defined as follows:

Volumetric Strain:
The volumetric strain is the unit change in volume, i.e. the change in volume divided by the original volume.

4.3.1 Two-Dimensional Volumetric Strain

Analogous to Eqn 3.5.1, the strain invariants are

$$\begin{aligned} I_1 &= \varepsilon_{xx} + \varepsilon_{yy} \\ I_2 &= \varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}^2 \end{aligned} \qquad \textbf{Strain Invariants} \qquad (4.3.1)$$

Using the strain transformation formulae, Eqns. 4.2.2, it will be verified that these quantities remain unchanged under any rotation of axes.

The first of these has a very significant physical interpretation. Consider the deformation of the material element shown in Fig. 4.3.1a. The volumetric strain is

$$\begin{aligned} \frac{\Delta V}{V} &= \frac{(a+\Delta a)(b+\Delta b)-ab}{ab} \\ &= (1+\varepsilon_{xx})(1+\varepsilon_{yy})-1 \\ &= \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{xx}\varepsilon_{yy} \end{aligned} \qquad (4.3.2)$$

If the strains are small, the term $\varepsilon_{xx}\varepsilon_{yy}$ will be very much smaller than the other two terms, and the volumetric strain in that case is given by

$$\frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} \qquad \textbf{Volumetric Strain} \qquad (4.3.3)$$

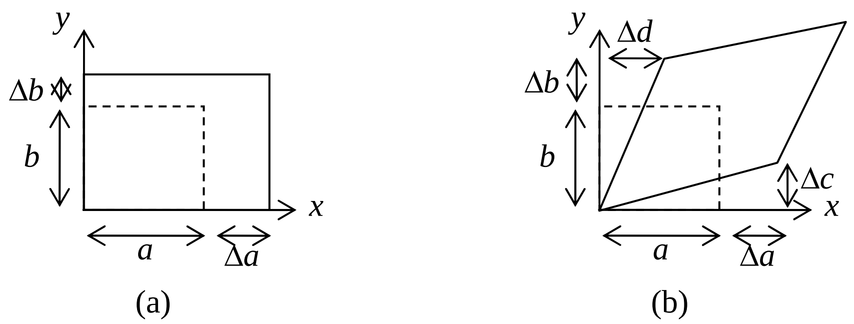

Figure 4.3.1: deformation of a material element; (a) normal deformation, (b) with shearing

Since by Eqn. 4.3.1 the volume change is an invariant, the normal strains in any coordinate system may be used in its evaluation. This makes sense: the volume change cannot depend on the particular axes we choose to measure it. In particular, the principal strains may be used:

$$\frac{\Delta V}{V} = \varepsilon_1 + \varepsilon_2 \qquad (4.3.4)$$

The above calculation was carried out for stretching in the *x* and *y* directions, but the result is valid for any arbitrary deformation. For example, for the general deformation shown in Fig. 4.3.1b, some geometry shows that the volumetric strain is $\Delta V / V = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}^2$, which again reduces to Eqns 4.3.3, 4.3.4, for small strains.

An important consequence of Eqn. 4.3.3 is that *normal strains induce volume changes*, whereas *shear strains induce a change of shape but no volume change*.

4.3.2 Three Dimensional Volumetric Strain

A slightly different approach will be taken here in the three dimensional case, so as not to simply repeat what was said above, and to offer some new insight into the concepts.

Consider the element undergoing strains $\varepsilon_{xx}, \varepsilon_{xy}$, etc., Fig. 4.3.2a. The same deformation is viewed along the principal directions in Fig. 4.3.2b, for which only normal strains arise.

The volumetric strain is:

$$\begin{aligned} \frac{\Delta V}{V} &= \frac{(a+\Delta a)(b+\Delta b)(c+\Delta c) - abc}{abc} \\ &= (1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3) - 1 \\ &\approx \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \end{aligned} \qquad (4.3.5)$$

and the squared and cubed terms can be neglected because of the small-strain assumption.

Since any elemental volume such as that in Fig. 4.3.2a can be constructed out of an infinite number of the elemental cubes shown in Fig. 4.3.3b (as in Fig. 4.2.7), this result holds for any elemental volume irrespective of shape.

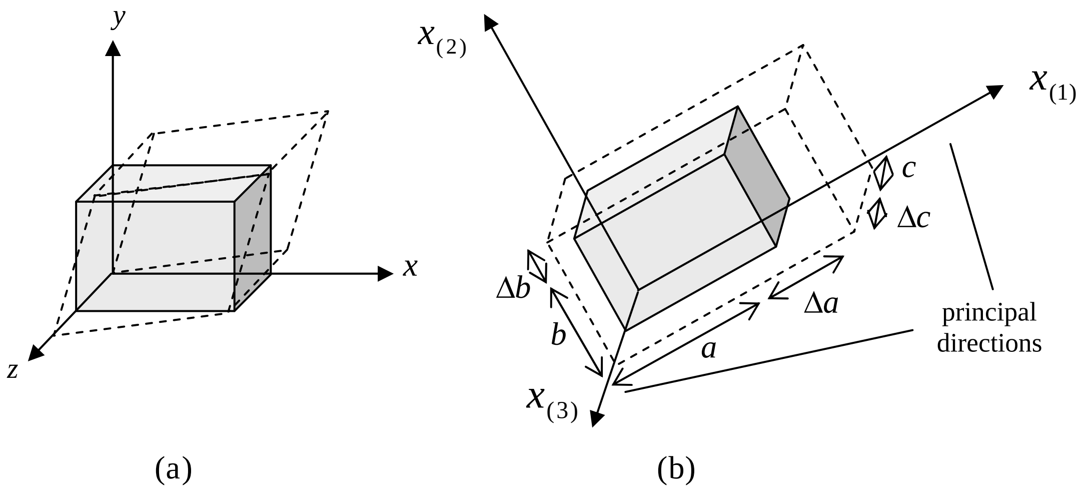

Figure 4.3.2: A block of deforming material; (a) subjected to an arbitrary strain; (a) principal strains