An improved integer programming formulation for indoor base station placement

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Abstract: An improved integer programming formulation is presented for the optimisation of indoor wireless base station placement in an interference-limited environment. Unlike the original, this formulation does not require the estimation of the loading level of each base station and hence can provide a more versatile solution.

Introduction: Base station placement is an important aspect of indoor wireless system planning. In order to minimise cost and reduce interference, the number of base stations used and their respective locations must be selected carefully. A generally accepted performance measure for a wireless system is the signal-to-interference ratio (SIR) [1, pp417-425], which is propagation path length dependent. Most published wireless system optimisation approaches, e.g. [2-5], have used non-linear or heuristic methods to solve the base station placement problem. In contrast, an alternative approach using integer programming (IP) was reported in [6]. This approach allows non-linear performance measures (such as SIR) to be considered given appropriate estimation of the loading levels on each base station (i.e. the number of users served by each base station). In this letter, an improved formulation that does not require a priori estimation of the loading level is reported.

Goal of optimisation: There are only a finite number of $B$ potential base station sites inside a given building. Similarly, it is assumed that there are $U$ locations where wireless users might reside. Thus, the goal is to select the minimum number of base stations from a set of given potential sites to provide services to potential user locations. In general, a base station configuration is considered feasible if two requirements are satisfied, namely:

1. Signal quality requirement - The forward link SIR of all users must exceed a minimum level $Q_{db}$;
2. Coverage requirement - All users must be able to communicate with a base station with received signals stronger than a specified threshold $P_{\text{min}}$.

Note that, if we assume the transmission power at each potential base station is fixed and known, the received signal strength $P_{bu}$ (where $b = 1, 2, 3 \ldots B$, $u = 1, 2, 3 \ldots U$) at each wireless user location from each potential base station site can be obtained using measurements (or a propagation model). If $B^*$ is the set of base stations being used, $G_p$ is the processing gain of the system and $L_b$ is the loading on (the number of users assigned to) a base station, $b$, the forward link SIR (in dB) from this base station to user location $u$ is,

$$SIR_{bu} = 10 \log_{10} \left( \frac{G_p P_{bu}}{L_b} + \frac{\left(L_b - 1\right) P_{u}}{L_b} \right) - 1.$$  \hfill (1)

**Original formulation:** The base station placement problem is formulated as a binary integer programming (BIP) problem [6]. The decision variables are:

- $X_{bu} = 1$ (0) if a link is (is not) established between base station $b$ and wireless user location $u$,
- $Y_b = 1$ (0) if a base station is (is not) deployed at potential site $b$,

for $b = 1, 2, 3 \ldots B; \quad u = 1, 2, 3 \ldots U$.

This formulation requires the user to estimate the loading of base station $b$, $L_b$, in order to calculate the SIR for the forward link. To meet requirement 1 (SIR requirement), a constraint is set to ensure that the desired signal is $Q$ times stronger than the sum of the interference (i.e. protection ratio $Q_{\text{dB}} = 10 \log_{10}(Q)$ dB). Thus, for each wireless user location $u$, we can formulate a constraint, namely,

$$\frac{G_p}{L_b} \left( \sum_{b} P_{bu} X_{bu} \right) \geq Q \left( \sum_{b} \left( P_{bu} Y_b - X_{bu} + \frac{L_b - 1}{L_b} P_{bu} X_{bu} \right) \right) \quad u = 1, 2, 3 \ldots U.$$  \hfill (2)
To ensure validity of the solution, another constraint is added to restrict base station $b$ to serving no more than $L_b$ active users, i.e.,

$$\sum_{u=1}^{U} (X_{bu}) \leq L_b Y_b \quad b = 1,2,3...B. \quad (3)$$

To meet the coverage requirement, each wireless user should be served by one base station (in a non-diversity system), and their received signal strength from the desired base station must be above the receiver specified threshold $P_{min}$ after accounting for the effects of propagation and processing gain. Therefore, for any user location $u$,

$$\sum_{b=1}^{B} X_{bu} = 1 \quad u = 1,2,3...U, \quad \text{and} \quad (4)$$

$$G \sum_{b=1}^{B} P_{bu} X_{bu} \geq P_{min} \quad u = 1,2,3...U. \quad (5)$$

Finally, the objective is to minimise the required number of base stations, i.e. $\text{Minimise} \left( \sum_{b=1}^{B} Y_b \right)$.

**New formulation**: The new formulation includes an extra dimension that relates to the loading of each base station. Assuming the maximum base station loading is $L$, the new decision variables are defined as:

- $X_{bu} = 1 \ (0)$ if a link is (is not) established between base station $b$ and user location $u$ and there are a total of $l$ users communicating with this base station,

- $Y_{bl} = 1 \ (0)$ if a base station is (is not) deployed at potential site $b$ with a loading of $l$ users,

for $b = 1,2,3...B$; $u = 1,2,3...U$; $l = 1,2,3...L$.

To satisfy the forward link SIR requirements in this formulation, the constraint in (2) is modified. For each wireless user location $u$,
To validate the loading at each base station, two constraints are needed, namely, 1) to ensure only one loading level is chosen for each base station; and 2) to ensure that the number of wireless users communicating to the base station agrees with the loading level selected. These constraints are given by

\[
\sum_{i=1}^{L} (Y_{b,i}) \leq 1 \quad b = 1,2,3...B; \quad \text{and} \quad \sum_{i=1}^{L} (X_{bui}) = Y_{b,i} \cdot l \quad b = 1,2,3...B; \quad l = 1,2,3...L \quad \text{respectively.} \tag{7}
\]

The formulation for the coverage requirement is similar to that in the original formulation (4) and (5), but these are modified to become

\[
\sum_{b=1}^{B} \sum_{i=1}^{L} X_{bui} = 1 \quad u = 1,2,3...U, \quad \text{and} \quad \sum_{b=1}^{B} \sum_{i=1}^{L} P_{bw} X_{bui} \geq P_{\text{min}} \quad u = 1,2,3...U \quad \text{respectively.} \tag{9}
\]

Finally, the objective function is modified to \( \text{Minimise} \{ \sum_{b=1}^{B} \sum_{i=1}^{L} Y_{b,i} \}. \)

**Example:** For the indoor environment in Fig. 1, we aim to select a number of base station sites from a group of 12 potential sites to provide services to 27 wireless users in a CDMA system. In this example, the processing gain is assumed to be 128 (\( G_p = 128 \)). The receiver sensitivity is assumed to be -90dBm (\( P_{\text{min}} = 1pW \)) and a protection ratio of 8.45dB (\( Q = 7 \)) is used. All antennas are assumed to be omni-directional and all the base stations transmit at \( P_t = 1mW \). For simplicity of illustration, the loading level \( L_b \) is assumed to be the same for all base stations. For an operating frequency of 1.8GHz, a database of path loss data has been obtained via measurements. To demonstrate the influence
that the loading level has on the original formulation, the value of $L_b$ is varied between 10, 15 and 20. These results are compared to those delivered by the new formulation. All results presented in this letter were computed using the commercial optimisation tool AMPL/CPLEX.

**Results:** Table 1 summarises the results given by the two different IP formulations. Each solution includes the base station sites to be used and the minimum and maximum SIR values achieved at the 27 potential user locations. As with any IP optimisation, the results show that the constraints (SIR and coverage) are satisfied whenever a solution is found. However, if too large a value of $L_b$ is selected using the original formulation, (as shown for the case where $L_b = 20$ in Table 1), no solution will be found because interference levels will be excessive. Conversely, if too small a value of $L_b$ is selected, the solution will be sub-optimal because each base station’s capacity is not fully exploited. Including the choice of $L_b$ in the new formulation yields the most efficient feasible solution.

**Conclusions:** In this letter, an improved wireless system planning approach that uses integer programming has been introduced. Unlike the original IP formulation, the improved formulation does not require *a priori* selection of the base station loading level and yet allows SIR to be used as a performance measure. Furthermore, the solutions generated are guaranteed to meet all specified constraints.
References:


Figure captions:

Fig. 1. Floor plan of the building used in the example problem (dimension 18.5m×18.5m)

O = users

+ = potential base station sites

Table 1. Solution summary of the base station configurations given by the two formulations
Figure 1

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Table 1.

<table>
<thead>
<tr>
<th>Formulation ($L_\alpha$)</th>
<th>Base station sites (number of users)</th>
<th>User location with maximum SIR (SIR (dB))</th>
<th>User location with minimum SIR (SIR (dB))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original (10)</td>
<td>1 (8), 7 (9), 11 (10)</td>
<td>1 (12.5)</td>
<td>21 (9.6)</td>
</tr>
<tr>
<td>Original (15)</td>
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<td>21 (10.3)</td>
<td>24 (9.3)</td>
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<td>No solution exists</td>
<td>No solution exists</td>
</tr>
<tr>
<td>New</td>
<td>5 (15), 12 (12)</td>
<td>5 (10.7)</td>
<td>12 (8.9)</td>
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