

1 Vectors & Tensors

The mathematical modeling of the physical world requires knowledge of quite a few different mathematics subjects, such as Calculus, Differential Equations and Linear Algebra. These topics are usually encountered in fundamental mathematics courses. However, in a more thorough and in-depth treatment of mechanics, it is essential to describe the physical world using the concept of the **tensor**, and so we begin this book with a comprehensive chapter on the tensor.

The chapter is divided into three parts. The first part covers vectors (§1.1-1.7). The second part is concerned with second, and higher-order, tensors (§1.8-1.15). The second part covers much of the same ground as done in the first part, mainly generalizing the vector concepts and expressions to tensors. The final part (§1.16-1.19) (not required in the vast majority of applications) is concerned with generalizing the earlier work to curvilinear coordinate systems.

The first part comprises basic vector algebra, such as the dot product and the cross product; the mathematics of how the components of a vector transform between different coordinate systems; the symbolic, index and matrix notations for vectors; the differentiation of vectors, including the gradient, the divergence and the curl; the integration of vectors, including line, double, surface and volume integrals, and the integral theorems.

The second part comprises the definition of the tensor (and a re-definition of the vector); dyads and dyadics; the manipulation of tensors; properties of tensors, such as the trace, transpose, norm, determinant and principal values; special tensors, such as the spherical, identity and orthogonal tensors; the transformation of tensor components between different coordinate systems; the calculus of tensors, including the gradient of vectors and higher order tensors and the divergence of higher order tensors and special fourth order tensors.

In the first two parts, attention is restricted to rectangular Cartesian coordinates (except for brief forays into cylindrical and spherical coordinates). In the third part, curvilinear coordinates are introduced, including covariant and contravariant vectors and tensors, the metric coefficients, the physical components of vectors and tensors, the metric, coordinate transformation rules, tensor calculus, including the Christoffel symbols and covariant differentiation, and curvilinear coordinates for curved surfaces.

