

## Answers to Selected Problems: Chapter 2

### 2.1

1. (b)  $\rho = 3x_1 + x_2 - t$   
(c) No  
(d)  $x_1 = X_1, x_2 = X_2 + t, x_3 = X_3 + t$

### 2.2

3.  
(a)

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -2 & 2 & 1 \end{bmatrix}, \quad \mathbf{C} = \mathbf{F}^T \mathbf{F} = \begin{bmatrix} 5 & -4 & 0 \\ -4 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

- (b)  $d\mathbf{x}^{(1)} \cdot d\mathbf{x}^{(2)} = -4$ , not perpendicular

- (c)  $\lambda^2 = 1$

- (d)  $\mathbf{E} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 & -4 & 0 \\ -4 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}, \mathbf{e} = \frac{1}{9} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

- (e) Principal values of  $\mathbf{C}$  are 1,9,9; principal values of  $\mathbf{U}$  are 1,3,3.

$$\mathbf{R} = \frac{1}{3} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \\ -2 & 2 & 1 \end{bmatrix}$$

### 2.4

1. (a)  $-\frac{2e^{-2t}}{(x_1^2 + x_2^2 + x_3^2)^2} [x_1^2 + x_2^2 + x_3^2 - x_1x_2 + 3x_1x_3 + 3x_2x_3]$

### 2.5

2.  $\mathbf{l} = \begin{bmatrix} 2x_1x_2 & x_1^2 & 0 \\ 0 & 4x_2x_3 & 2x_2^2 \\ 3x_2x_3 & 3x_1x_3 & 3x_1x_2 \end{bmatrix}, \quad \mathbf{e}_1 \mathbf{l} \mathbf{e}_1 = 2x_1x_2$

5.  $\mathbf{v}(\mathbf{x}, t) = \left(\frac{1}{t}x_1 - t\right)\mathbf{e}_1 + \left(-\frac{1}{3t^2}x_1 + \frac{1}{t}x_2 - \frac{1}{3}\right)\mathbf{e}_2 + \frac{1}{3}x_3\mathbf{e}_3, \quad \operatorname{div} \mathbf{v} = \frac{3}{t}$