Answers to Selected Problems: Part II, Chapter 3

3.1

1.
$$\frac{E}{2(1-v^2)}\left[2\frac{\partial^2 u_x}{\partial x^2} + (1+v)\frac{\partial^2 u_y}{\partial x \partial y} + (1-v)\frac{\partial^2 u_x}{\partial y^2}\right] + b_x = 0$$

$$\frac{E}{2(1-v^2)}\left[2\frac{\partial^2 u_y}{\partial y^2} + (1+v)\frac{\partial^2 u_x}{\partial x \partial y} + (1-v)\frac{\partial^2 u_y}{\partial x^2}\right] + b_y = 0$$

2. $\sigma_{xx} = \dfrac{2AE}{(1+v)^2}x, \quad = \dfrac{2AE}{(1+v)^2}(2+v)x, \quad \sigma_{xy} = -\dfrac{2AE}{(1+v)^2}y$

3.   (a) Yes, (b) Yes, (c) u_x, u_y non-zero along the base (ε_{xx} is non-zero which in itself is inconsistent with $u_x = 0$ along base)

3.2

3. $A = 5B$, $\dfrac{\Delta V}{V} = \dfrac{B}{E}\left[(1-2v)(30xy^2 - 10x^3)\right]$

4. $\varepsilon_{xx} = \frac{5}{4E}\left[15x^2 y - 17y^3\right], \varepsilon_{yy} = \frac{5}{4E}\left[3x^2 y + 11y^3\right], \varepsilon_{xy} = \frac{5}{4E}\left[-4x^3 - 24xy^2\right]$

   $u_x = \frac{5}{4E}\left[5x^3 y - 17y^3 x\right] + Cy + B, u_y = \frac{5}{4E}\left[-\frac{13}{4}x^4 + \frac{3}{2}x^2 y^2 + \frac{11}{4}y^4\right] + Cx + A$

6. $u_x = \dfrac{F}{4Eb}\left[3\dfrac{x^2 y}{b^2} - (2+v)\dfrac{y^3}{b^2} + 6(1+v)y - 3\dfrac{L^2}{b^2}y\right]$

   $u_y = \dfrac{F}{4Eb}\left[-3v\dfrac{xy^2}{b^2} - \dfrac{x^3}{b^2} + 3\dfrac{L^2}{b^2}x - 2\dfrac{L^3}{b^2}\right]$

8. $\sigma_{xx} = -\rho gy, \quad \sigma_{yy} = \dfrac{\rho g}{\tan^2 \beta}\left(-\dfrac{2}{\tan \beta}x + y\right), \quad \sigma_{xy} = -\dfrac{\rho g}{\tan^2 \beta}x$