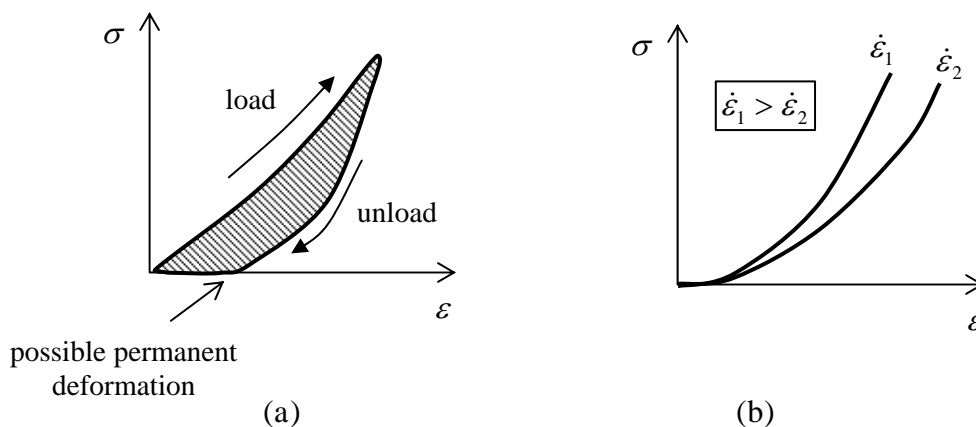


## 10.1 The Response of Viscoelastic Materials

### 10.1.1 Viscoelastic Materials

The basic response of the viscoelastic material was discussed in section 5.3.2. Repeating what was said there, the typical response of a viscoelastic material is as sketched in Fig. 10.1.1. The following will be noted:

- (i) the loading and unloading curves do not coincide, Fig. 10.1.1a, but form a hysteresis loop
- (ii) there is a dependence on the rate of straining  $d\varepsilon/dt$ , Fig. 10.1.1b; the faster the stretching, the larger the stress required
- (iii) there may or may not be some permanent deformation upon complete unloading, Fig. 10.1.1a



**Figure 10.1.1: Response of a Viscoelastic material in the Tension test; (a) loading and unloading with possible permanent deformation (non-zero strain at zero stress), (b) different rates of stretching**

The effect of *rate* of stretching shows that the viscoelastic material *depends on time*. This contrasts with the elastic material, whose constitutive equation is independent of time, for example it makes no difference whether an elastic material is loaded to some given stress level for one second or one day, or loaded slowly or quickly; the resulting strain will be the same.

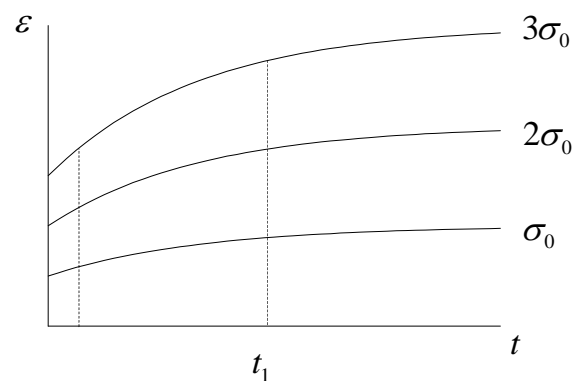
It was shown in Chapter 5 that the area beneath the stress-strain curve is the energy per unit volume; during loading, it is the energy stored in a material, during unloading it is the energy recovered. There is a difference between the two for the viscoelastic material, indicated by the shaded region in Fig. 10.1.1a. This shaded region is a measure of the energy lost through heat transfer mechanisms during the deformation.

Most engineering materials undergo plasticity, meaning permanent deformations occur once the stress goes above the elastic limit. The stress-strain curve for these materials can look very similar to that of Fig. 10.1.1a, but, in contrast to viscoelasticity, plasticity is rate independent. Plasticity will be discussed in chapter 11.

## Linear Viscoelasticity

**Linear viscoelastic** materials are those for which there is a linear relationship between stress and strain (at any given time),  $\sigma \propto \varepsilon$ . As mentioned before, this requires also that the strains are small, so that the engineering strain measure can be used (since the exact strain is inherently non-linear).

Strain-time curves for a linear viscoelastic material subjected to various constant stresses are shown in Fig. 10.1.2. At any given time, say  $t_1$ , the strain is proportional to stress, so that the strain there due to  $3\sigma_0$  is three times the strain due to  $\sigma_0$ .



**Figure 10.1.2: Strain as a function of time at different loads**

Linear viscoelasticity is a reasonable approximation to the time-dependent behaviour of metals and ceramics at relatively low temperatures and under relatively low stress. However, its most widespread application is in the modelling of polymers.

### 10.1.2 Testing of Viscoelastic Materials

The tension test described in section 5.2 is the standard materials test. A number of other tests which are especially useful for the characterisation of viscoelastic materials have been developed, and these are discussed next.

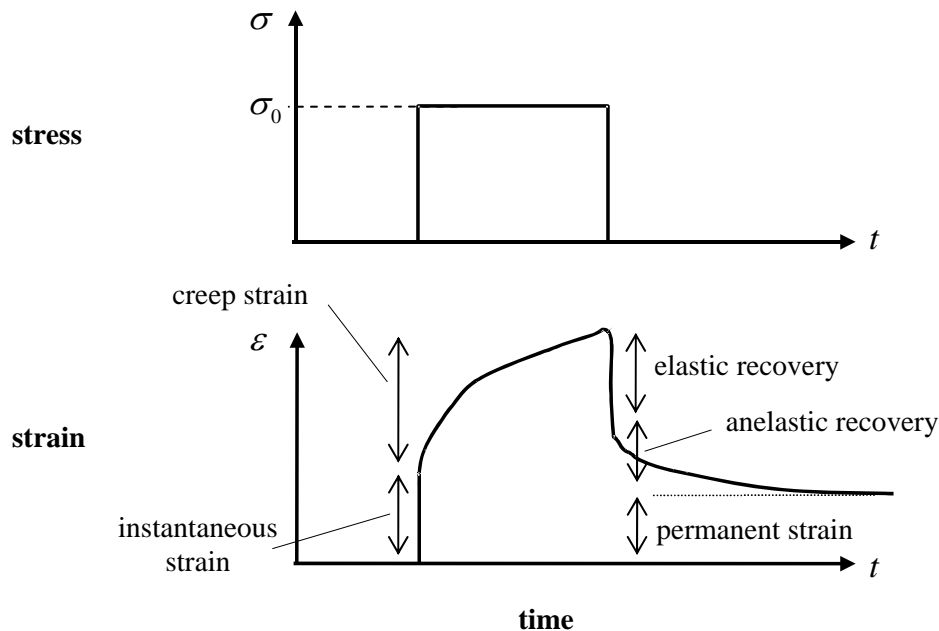
#### The Creep and Recovery Test

The **creep-recovery test** involves loading a material at constant stress, holding that stress for some length of time and then removing the load. The response of a typical viscoelastic material to this test is shown in Fig. 10.1.3.

First there is an instantaneous elastic straining, followed by an ever-increasing strain over time known as **creep strain**. The creep strain usually increases with an ever decreasing strain rate so that eventually a more-or-less constant-strain steady state is reached, but many materials often do not reach such a noticeable steady-state, even after a very long time.

When unloaded, the elastic strain is recovered immediately. There is then **anelastic** recovery – strain recovered over time; this anelastic strain is usually very small for metals, but may be significant in polymeric materials. A permanent strain may then be left in the material<sup>1</sup>.

A test which focuses on the loading phase only is simply called the **creep test**.

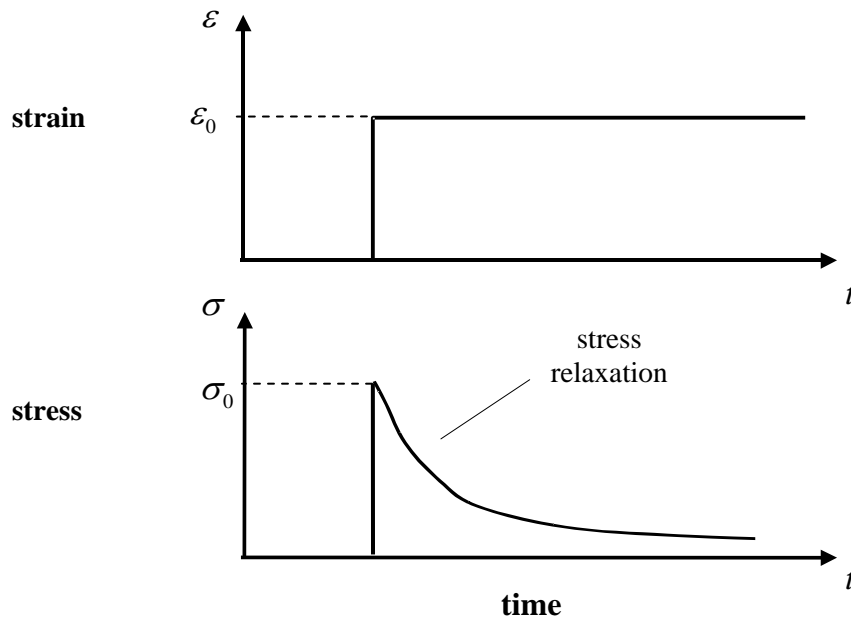


**Figure 10.1.3: Strain response to the creep-recovery test**

### Stress Relaxation Test

The stress relaxation test involves straining a material at constant strain and then holding that strain, Fig. 10.1.4. The stress required to hold the viscoelastic material at the constant strain will be found to decrease over time. This phenomenon is called **stress relaxation**; it is due to a re-arrangement of the material on the molecular or micro-scale.

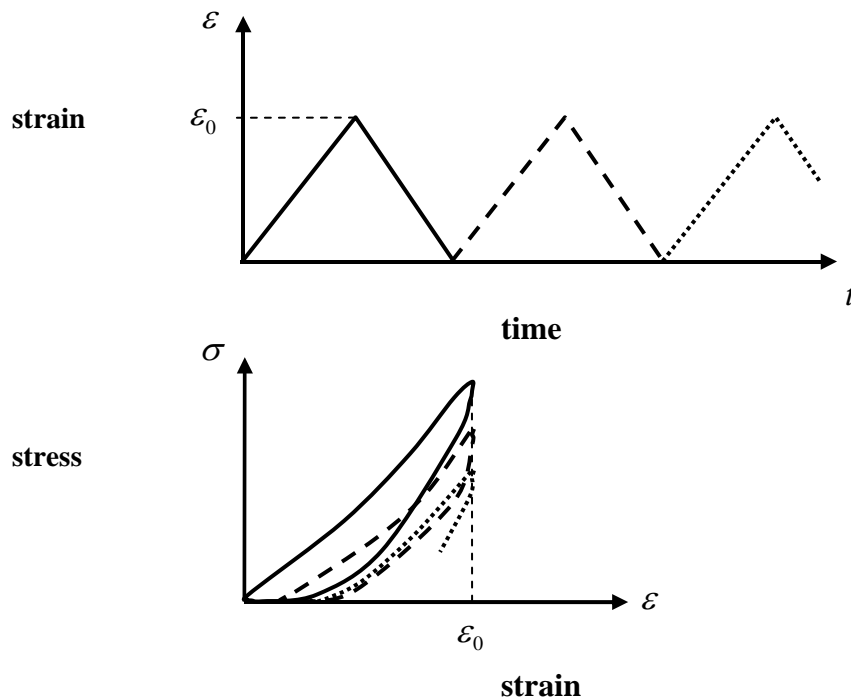
<sup>1</sup> if the load is above the yield stress, then some of the permanent deformation will be instantaneous plastic (rate-independent) strain; the subject of this chapter is confined to materials which are loaded up to a stress below any definable yield stress; rate-dependent materials with a yield stress above which permanent deformation take place, the viscoplastic materials, are discussed in Chapter 12



**Figure 10.1.4: Stress response to the stress-relaxation test**

### The Cyclic Test

The cyclic test involves a repeating pattern of loading-unloading, Fig. 10.1.5 (see section 5.2.5). It can be strain-controlled (with the resulting stress observed), as in Fig. 10.1.5, or stress-controlled (with the resulting strain observed). The results of a cyclic test can be quite complex, due to the creep, stress-relaxation and permanent deformations.



**Figure 10.1.5: Typical stress response to the cyclic test**