3.5b Stress Boundary Conditions: Continued

Consider now in more detail a surface between two different materials, Fig. 3.5.16. One says that the normal and shear stresses are **continuous** across the surface, as illustrated.

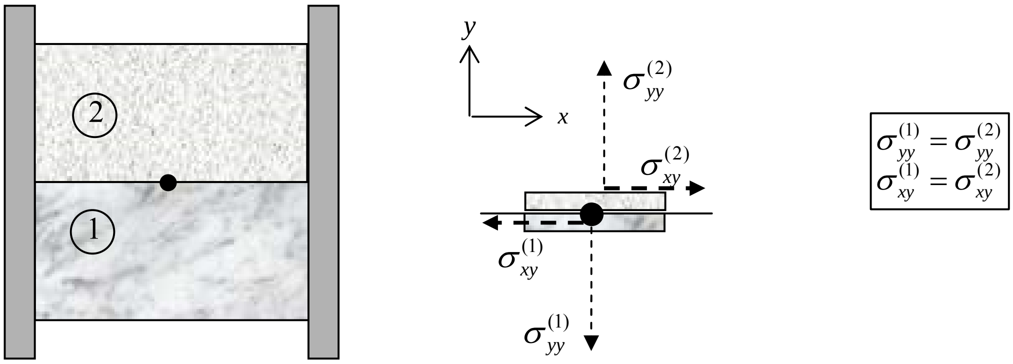


Figure 3.5.16: normal and shear stress continuous across an interface between two different materials, material '1' and material '2'

Note also that, since the shear stress σ_{xy} is the same on both sides of the surface, the shear stresses acting on both sides of a perpendicular plane passing *through* the interface between the materials, by the symmetry of stress, must also be the same, Fig. 3.5.17a.

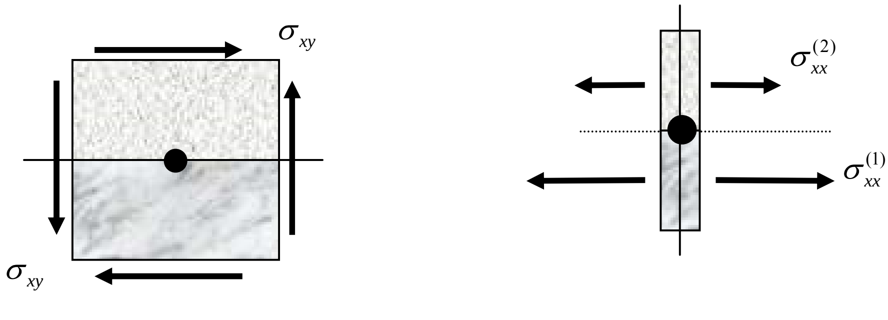


Figure 3.5.17: stresses at an interface; (a) shear stresses continuous across the interface, (b) tangential stresses not necessarily continuous

However, again, the tangential stresses, those acting parallel to the interface, do *not* have to be equal. For example, shown in Fig. 3.5.17b are the tangential stresses acting in the upper material, $\sigma_{xx}^{(2)}$ - they balance no matter what the magnitude of the stresses $\sigma_{xx}^{(1)}$.

Description of Boundary Conditions

The following example brings together the notions of stress boundary conditions, stress components, equilibrium and equivalent forces.

Example

Consider the plate shown in Fig. 3.5.18. It is of width $2a$, height b and depth t. It is subjected to a tensile stress r, pressure p and shear stresses s. The applied stresses are uniform through the thickness of the plate. It is welded to a rigid base.

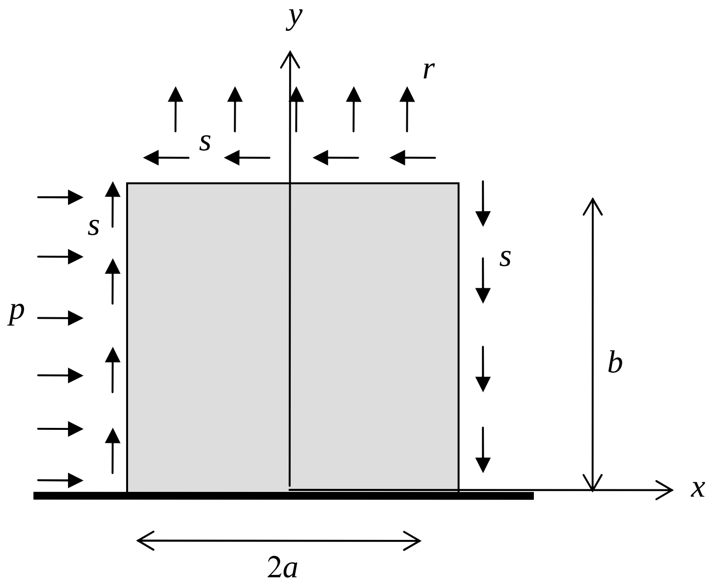


Figure 3.5.18: a plate subjected to stress distributions

Using the $x-y$ axes shown, the stress boundary conditions can be expressed as:

Left-hand surface: $\begin{cases} \sigma_{xx}(-a,y) = -p \\ \sigma_{xy}(-a,y) = -s \end{cases}, \quad 0 < y < b$

Top surface: $\begin{cases} \sigma_{yy}(x,b) = +r \\ \sigma_{xy}(x,b) = -s \end{cases}, \quad -a < x < +a$

Right-hand surface: $\begin{cases} \sigma_{xx}(+a,y) = 0 \\ \sigma_{xy}(+a,y) = -s \end{cases}, \quad 0 < y < b$

Note carefully the description of the normal and shear stresses over each side and the signs of the stress components.

The stresses at the lower edge are unknown (there is a displacement boundary condition there: zero displacement). They will in general not be uniform. Using the given $x-y$ axes, these unknown reaction stresses, exerted by the base on the plate, are (see Fig 3.5.19)

Lower surface: $\begin{cases} \sigma_{yy}(x,0) \\ \sigma_{xy}(x,0) \end{cases}, \qquad -a < x < +a$

Note the directions of the arrows in Fig. 3.5.19, they have been drawn in the direction of positive $\sigma_{yy}(x,0)$, $\sigma_{xy}(x,0)$.

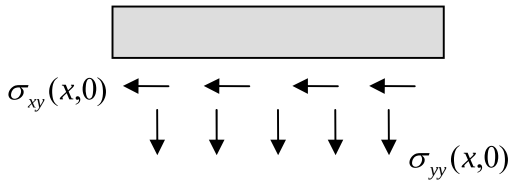


Figure 3.5.19: unknown reaction stresses acting on the lower edge

For force equilibrium of the complete plate, consider the free-body diagram 3.5.20; shown are the resultant forces of the stress distributions. Force equilibrium requires that

$$\sum F_x = bpt - 2ast - t\int_{-a}^{+a} \sigma_{xy}(x,0)dx = 0$$

$$\sum F_y = 2art - t\int_{-a}^{+a} \sigma_{yy}(x,0)dx = 0$$

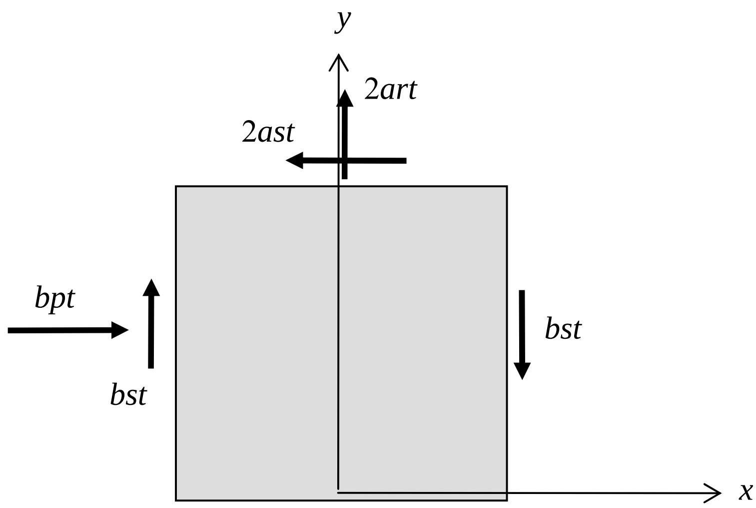


Figure 3.5.20: a free-body diagram of the plate in Fig. 3.5.18 showing the known resultant forces (forces on the lower boundary are not shown)

For moment equilibrium, consider the moments about, for example, the lower left-hand corner. One has

$$\sum M_0 = -bpt(b/2) + 2ast(b) + 2art(a) - bst(2a) - t\int_{-a}^{+a} \sigma_{yy}(x,0) \times (a + x)dx = 0$$

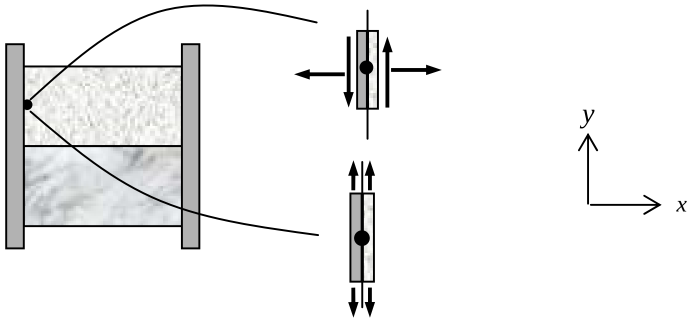
If one had taken moments about the top-left corner, the equation would read

$$\sum M_0 = +bpt(b/2) + 2art(a) - bst(2a)$$
$$- t\int_{-a}^{+a} \sigma_{xy}(x,0) \times b\,dx - t\int_{-a}^{+a} \sigma_{yy}(x,0) \times (a + x)dx = 0$$

■

Problems

8. Consider the point shown below, at the boundary between a wall and a dissimilar material. Label the stress components displayed using the coordinate system shown. Which stress components are continuous across the wall/material boundary? (Add a superscript 'w' for the stresses in the wall.)



9. A thin metal plate of width $2b$, height h and depth t is loaded by a pressure distribution $p(x)$ along $-a < x < +a$ and welded at its base to the ground, as shown in the figure below. Write down expressions for the stress boundary conditions (two on each of the three edges). Write down expressions for the force equilibrium of the plate and moment equilibrium of the plate about the corner A.