

2.2 The Statics of Particles

2.2.1 Equilibrium of a Particle

The statics of particles is the study of particles *at rest* under the action of forces. This situation is referred to as **equilibrium**, which is defined as follows:

Equilibrium of a Particle

A particle is in equilibrium when the resultant of all the forces acting on that particle is zero

In practical problems, one will want to introduce a coordinate system to describe the action of forces on a particle. It is important to note that a force exists independently of any coordinate system one might use to describe it. For example, consider the force \mathbf{F} in Fig. 2.2.1. Using the vector polygon law, this force can be decomposed into combinations of any number of different individual forces; these individual forces are referred to as **components** of \mathbf{F} . In particular, shown in Fig 2.2.1 are three cases in which \mathbf{F} is decomposed into two rectangular (perpendicular) components, the components of \mathbf{F} in “direction x ” and in “direction y ”, \mathbf{F}_x and \mathbf{F}_y .

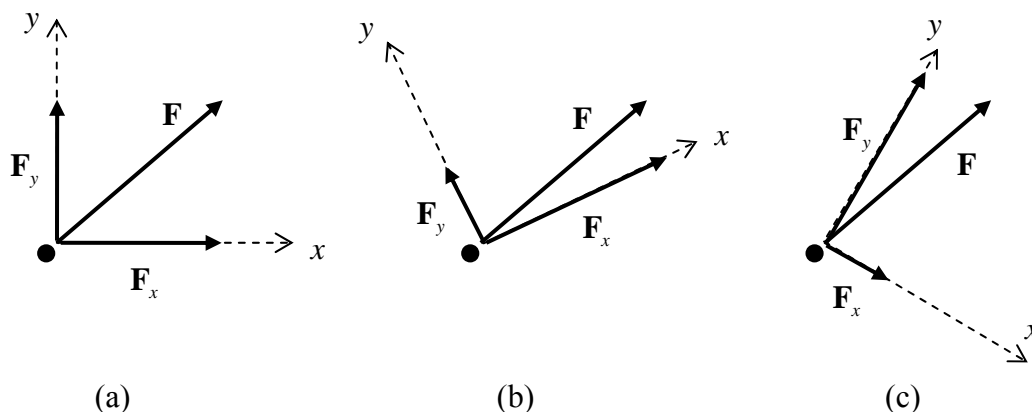


Figure 2.2.1: A force \mathbf{F} decomposed into components \mathbf{F}_x and \mathbf{F}_y using three different coordinate systems

By resolving forces into rectangular components, one can obtain analytic solutions to problems, rather than relying on graphical solutions to problems, for example as done in Fig. 2.1.2. In order that the resultant force \mathbf{F} on a body be zero, one must have that the resultant force in the x and y directions are zero individually¹, as illustrated in the following example.

¹ and in the z direction if one is considering a three dimensional problem

Example

Consider the particle in Fig. 2.2.2, subjected to forces \mathbf{F}_1 , \mathbf{F}_2 , \mathbf{F}_3 . The particle is in equilibrium and so by Newton's Laws the resultant force is zero, $\mathbf{F} = \mathbf{0}$. The forces are decomposed into horizontal and vertical components \mathbf{F}_{1x} , \mathbf{F}_{2x} , \mathbf{F}_{3x} and \mathbf{F}_{1y} , \mathbf{F}_{2y} , \mathbf{F}_{3y} . The horizontal forces may be added together to get a single horizontal force \mathbf{F}_x , which must equal zero. This force \mathbf{F}_x should be evaluated using the vector polygon law but, since the individual forces \mathbf{F}_{1x} , \mathbf{F}_{2x} , \mathbf{F}_{3x} all lie along the same line, one need only add together the *magnitudes* of these vectors, which involves simply an addition of *scalars*:

$F_{1x} + F_{2x} + F_{3x} = 0$. Similarly, one has $F_{1y} + F_{2y} + F_{3y} = 0$. These equations could be used to evaluate, for example, the force \mathbf{F}_1 , if only \mathbf{F}_2 and \mathbf{F}_3 were known.

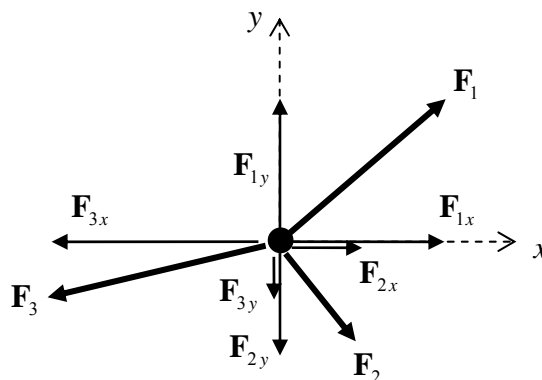


Figure 2.2.2: Calculating the resultant of three forces by decomposing them into horizontal and vertical components

■

In general then, if a set of forces $\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_n$ act on a particle, the particle is in equilibrium if and only if

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0 \quad \text{Equations of Equilibrium (particle)} \quad (2.2.1)$$

These are known as the **equations of equilibrium for a particle**. They are three equations and so can be used to solve problems involving three “unknowns”, for example the three components of one of the forces. In two-dimensional problems (as in the next example), they are a set of two equations.

Example

Consider the system of two cables attached to a wall shown in Fig. 2.2.3a. The cables meet at C, and this point is subjected to the two forces shown. Assume now that there are

forces arising in the cables AC and BC, indicated by the arrows in Fig. 2.2.3b². One can now draw a **free body diagram** of the particle C. The free body diagram concept is incredibly important and it is used in the most simple and in the most complex of problems, and will be used again and again in what follows. A free body diagram isolates a body (in this case the particle C) from its surroundings, and one considers all the forces, and *only* those forces, acting *on* that body, as shown in Fig 2.2.3b.

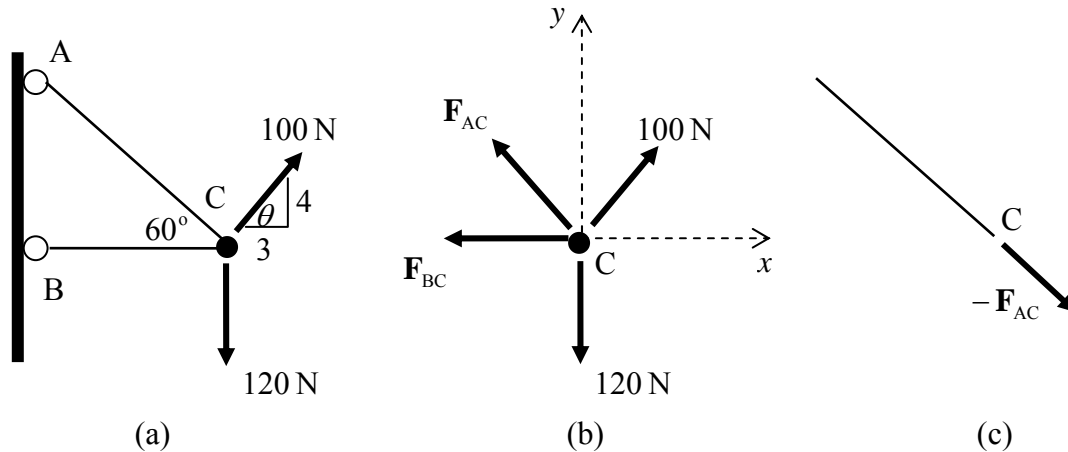


Figure 2.2.3: Calculating the tension in cables; (a) the cable system, (b) a free-body diagram of particle C, (c) cable AC in equilibrium

The equations of equilibrium for particle C are

$$\begin{aligned}\sum F_x &= -F_{BC} - F_{AC} \cos 60 + 100 \cos \theta = 0, \\ \sum F_y &= F_{AC} \cos 30 + 100 \sin \theta - 120 = 0\end{aligned}$$

leading to $F_{AC} = 46.2 \text{ N}$, $F_{BC} = 36.9 \text{ N}$.

The results are positive numbers; if the answer was negative, the arrow we assumed to be going towards C would in fact have been going the other way, away from C. We guessed right.

The cable exerts a tension/pulling force on particle C and so, from Newton's third law, C must exert an equal and opposite force on the cable, as illustrated in Fig. 2.2.3c. ■

2.2.2 Rough and Smooth Surfaces

Fig 2.2.4a shows a particle in equilibrium, sitting on a rough surface and subjected to a force \mathbf{F} . Such a surface is one where frictional forces are large enough to prevent tangential motion. The free body diagram of the particle is shown in Fig. 2.2.4b. The friction reaction force is \mathbf{R}_f (preventing movement along the surface) and the normal

² it does not matter which way you draw the arrows (away from C or towards C); if you do the calculation correctly, you will still get the same, correct, answer

reaction force is \mathbf{N} (preventing movement through the surface) and these lead to the resultant reaction force \mathbf{R} which, by Newton's Laws, must balance \mathbf{F} .

When a particle meets a smooth surface, there is no resistance to tangential movement. The particle is subjected to only a normal reaction force, and thus a particle in equilibrium can only sustain a purely normal force. This is illustrated in Fig. 2.2.4c.

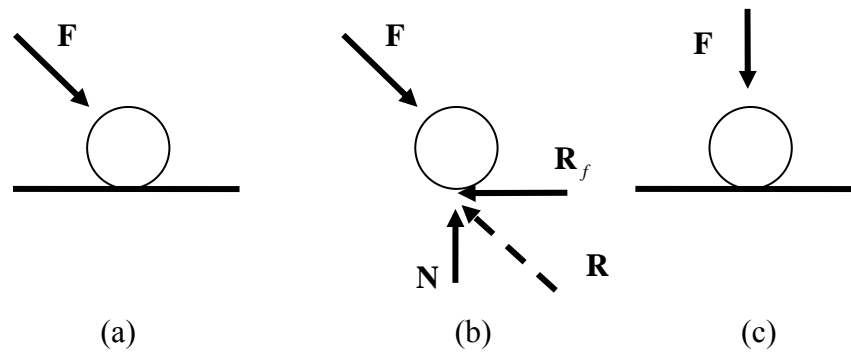
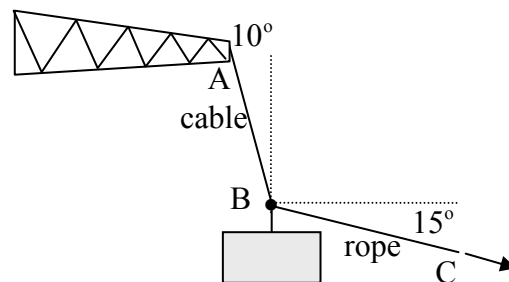


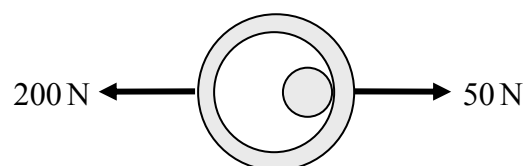
Figure 2.2.4: a particle sitting on a surface; (a) a rough surface, (b) a free-body diagram of the particle in (a), (c) a smooth surface

2.2.3 Problems

1. A 3000kg crate is being unloaded from a ship. A rope BC is pulled to position the crate correctly on the wharf. Use the Equations of Equilibrium to evaluate the tensions in the crane-cable AB and rope. [Hint: create a free body for particle B.]



2. A metal ring sits over a stationary post, as shown in the plan view below. Two forces act on the ring, in opposite directions. Draw a free body diagram of the ring including the reaction force of the post *on* the ring. Evaluate this reaction force. Draw a free body diagram of the post and show also the forces acting on it.



3. Two cylindrical barrels of radius 500mm are placed inside a container, a cross section of which is shown below. The mass of each barrel is 10kg. All surfaces are

smooth. Draw free body diagrams of each barrel, including the reaction forces exerted by the container walls *on* the barrels, the weight of each barrel, which can be assumed to act through the barrel centres, and the reaction forces of barrel on barrel. Apply the Equations of Equilibrium to each barrel. Evaluate all forces. What forces act on the container walls?

