Answers to Selected Problems: Part II, Chapter 8

8.1.6

2. 0.497% , 18.907%

6.

(i) $K=\dfrac{d\sigma}{d\varepsilon}=\left[\dfrac{1}{E}+n\dfrac{1}{b}\left(\varepsilon^{p}\right)^{1-1/n}\right]^{-1}$, $H=\dfrac{d\sigma}{d\varepsilon^{p}}=\left[n\dfrac{1}{b}\left(\varepsilon^{p}\right)^{1-1/n}\right]^{-1}$

(ii) $-300.848\times10^{6}\,\mathrm{Pa}$

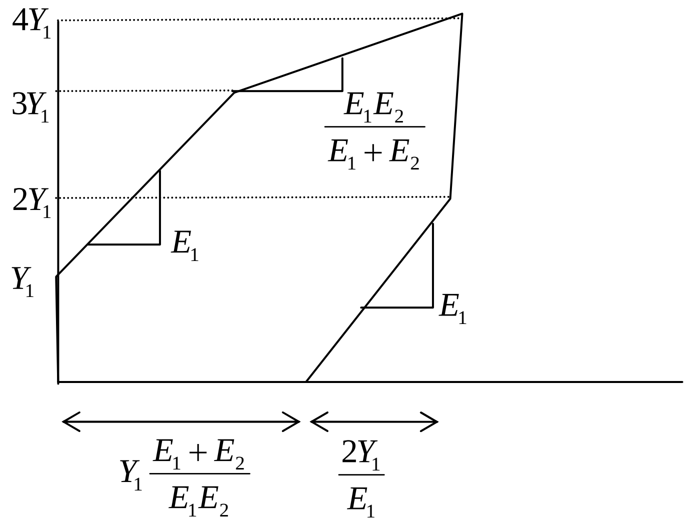
7.

(i) $E=E_1+E_2$

(ii) $\sigma=Y\dfrac{E_1+E_2}{E_1}$ at first yield.

(iii) $K=E_2$, $H=\dfrac{E_2}{E_1}\left(E_1+E_2\right)$

8.



8.2.3

1.

(a) $\sigma_m=\dfrac{\sigma_0}{3}$, $\mathbf{s}=\begin{bmatrix}\frac{2}{3}\sigma_0 & 0 & 0\\ 0 & -\frac{1}{3}\sigma_0 & 0\\ 0 & 0 & -\frac{1}{3}\sigma_0\end{bmatrix}$

(b) $\sigma_m=0$, $\mathbf{s}=\begin{bmatrix}0 & \tau & 0\\ \tau & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

2.

We have $\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$ so that

(a) 2

(b) $\mathbf{s} = \begin{bmatrix} -1 & 2 & 4 \\ 2 & 0 & 1 \\ 4 & 1 & 1 \end{bmatrix}$

(c) $J_2 = 22,\ J_3 = 13$

8.

$\sigma_{oct} = 2,\ \tau_{oct} = \sqrt{\frac{44}{3}} \approx 3.83$

8.3.6

1. (a) $Y_{crit} = 90$, (b) $Y_{crit} = 78.10$

5. Point D: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}$, Point E: $\begin{bmatrix} -\sigma & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6. $\tau = \frac{1}{2}Y$ (in which case the principal stresses are $\sigma_1 / Y = \frac{1+\sqrt{5}}{4},\ \sigma_2 / Y = -\frac{\sqrt{5}-1}{4}$)

7. B: $(-1,0,1)$, C: $(1,1,-2)$

11.

(a) $\rho = \frac{T_1 + 2T_2}{\sqrt{3}}$, $s = \sqrt{\frac{2}{3}}(T_1 - T_2)$, $\theta = 0$

(b) $\rho = -\frac{p_1 + 2p_2}{\sqrt{3}}$, $s = \sqrt{\frac{2}{3}}(p_1 - p_2)$, $\theta = 60$

8.4.6

6. No, you end up with the same expression

7. (ii) At first yield, $\sigma_{xx}^Y = \frac{1-\nu}{1-2\nu}Y, \quad \sigma_{yy}^Y = \frac{\nu}{1-2\nu}Y, \quad \varepsilon_{xx}^Y = \frac{1+\nu}{E}Y$

8.

(ii)

The strains and stresses at first yield are

$$\varepsilon_{xx}^{Y} = \frac{Y}{E}, \quad \varepsilon_{yy}^{Y} = \varepsilon_{zz}^{Y} = -\frac{\nu Y}{E}, \quad \varepsilon_{xy}^{Y} = 0$$

$$\sigma_{xx}^{Y} = Y, \quad \sigma_{xy}^{Y} = 0$$

(iii)

$$d\lambda = -\frac{3}{2E}\frac{d\sigma}{\sigma}$$

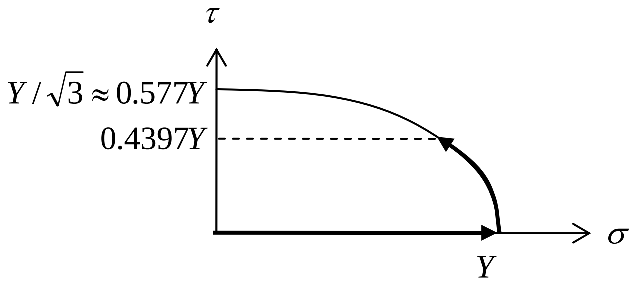
(iv)

$$d\varepsilon_{xy} = \frac{1+\nu}{E}d\tau + \frac{3}{2E}\frac{3\tau^2 d\tau}{Y^2 - 3\tau^2}$$

(vi)

$$\tau = Y\frac{1}{\sqrt{3}}\frac{e^2-1}{e^2+1} \approx 0.4397Y$$

The loading path is shown below:



8.6.8

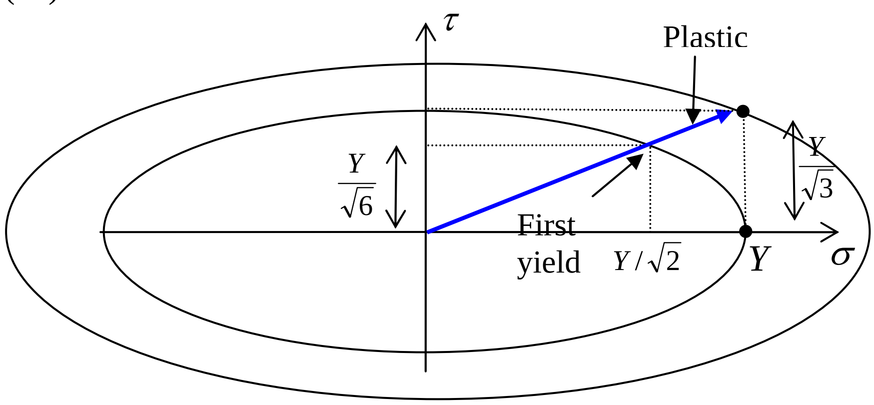
3.

(v-vi)

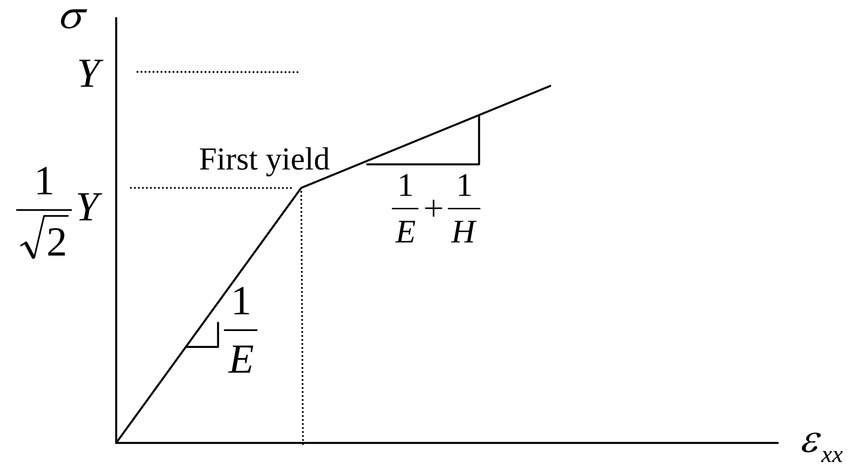
$$\varepsilon_{xx} = \left(\frac{1}{E} + \frac{1}{H}\right)\sigma - \frac{1}{\sqrt{2}}\frac{Y_0}{H}$$

$$\varepsilon_{xy} = \left(\frac{1+\nu}{\sqrt{3}E} + \frac{\sqrt{3}}{2}\frac{1}{H}\right)\sigma - \frac{\sqrt{3}}{2\sqrt{2}}\frac{Y_0}{H}$$

(vii)



(viii)



8.7.5

2. $\frac{\Delta V^p}{\Delta V} = d\lambda \frac{1}{2}(\alpha - 1)$

3. $\frac{\Delta V^p}{\Delta V} = d\lambda \frac{1}{2}(\beta - 1)$