Answers to Selected Problems: Chapter 8

8.1

1. $U = \dfrac{1}{3x_1^3} - \dfrac{1}{x_1}$

2. $W = mgh$, $W_{\text{grav}} = -mgh \equiv -U$

3.

$\dot{x} = -gt,\ x = -\dfrac{1}{2}gt^2 + h$

$W_{con} = -mg(x-h),\ \Delta K = \dfrac{1}{2}m\dot{x}^2$

$W_{con} = \Delta K = \dfrac{1}{2}mg^2t^2,\ U = +mg(x-h),\ 0 = \Delta U + \Delta K$.

4.

$\Delta K = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\dot{x}_0^2,\ W = -\left(\frac{1}{2}kx^2 - \frac{1}{2}kx_0^2\right),\ W = \Delta K,\ \Delta U = \frac{1}{2}kx^2 - \frac{1}{2}kx_0^2,$
$0 = \Delta U + \Delta K$.

5.

$\Delta K = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\dot{x}_0^2,\ W = F(x-x_0) - \left(\frac{1}{2}kx^2 - \frac{1}{2}kx_0^2\right) = -\dfrac{k}{2}(x-x_0)\left[x + x_0 - 2\dfrac{F}{k}\right]$

$W = \Delta K,\ \Delta U = \frac{1}{2}kx^2 - \frac{1}{2}kx_0^2,\ F(x-x_0) = \Delta U + \Delta K$.

6.

$\dot{x} = \left(\dfrac{F - F_{fr}}{m}\right)t + \dot{x}_0,\ \ x = \dfrac{1}{2}\left(\dfrac{F - F_{fr}}{m}\right)t^2 + \dot{x}_0 t + x_0$

$W = (F - F_{fr})(x - x_0) = (F - F_{fr})\left\{\dfrac{1}{2}\left(\dfrac{F - F_{fr}}{m}\right)t^2 + \dot{x}_0 t\right\}$

The energy dissipated: $H = F_{fr}(x - x_0) = -F_{fr}\left\{\dfrac{1}{2}\left(\dfrac{F - F_{fr}}{m}\right)t^2 + \dot{x}_0 t\right\}$

8.2

2. $U = 81.1\text{Nm}$

3. $\Delta = FL / AE$

5. $U_{\text{shear}} = \dfrac{3M_0^2}{5\mu AL},\ U_{\text{flex}} = \dfrac{5(\mu / E)L_2^3}{h^2 L} \times \left(\dfrac{3M_0^2}{5\mu AL} \right)$

6.

Rectangular cross-section: $U_{shear} = U_{flex} \left[\left(\dfrac{h}{L} \right)^2 \dfrac{E}{\mu} \right]$

Circular cross-section: $U_{shear} = U_{flex} \left[\dfrac{25}{36} \left(\dfrac{2r}{L} \right)^2 \dfrac{E}{\mu} \right]$

7. $\Delta = \dfrac{2FL}{Eh^2}$

8. $\Delta = \dfrac{wL^4}{8EI}$

9. $\dfrac{FL}{3EA}$

8.3

1. $C / U = 3$

8.5

2.

$\delta W = 10^{-8} EAL$

For the constant virtual displacement, the internal virtual work is zero.

3. $\Delta = \dfrac{LP}{A(E_1 + 2E_2)} = 0.5\text{mm}$

8.6

1. $u = \gamma(x^2 - 2Lx)$, $\gamma = -\dfrac{F}{4EAL} \quad \rightarrow \quad u = \dfrac{FL}{3EA}\left\{\dfrac{3}{4}\left[2\left(\dfrac{x}{L}\right) - \left(\dfrac{x}{L}\right)^2\right]\right\}$

2. $v = -\dfrac{2}{EI}\dfrac{L^3}{\pi^4}\left(\dfrac{2Lf}{\pi} + P\right)\sin\left(\pi\dfrac{x}{L}\right)$

3. $u = \dfrac{2}{3EA_0}\left(P + f\dfrac{L}{2}\right)x$